Course on “Advanced Statistical Physics”

School of Nano Science, for Research in Fundamental Sciences (IPM), Tehran, Iran
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◉ **Further information about this course, including problem sets and term-essay topics, are available at:** **http://physics.ipm.ac.ir/~naji/particles_SS2016.html**

◉ **Subjects covered in this course:**

✤ **Part I — Equilibrium Statistical Physics**

- **Foundations: Closed and quasi-closed systems**
  - Phase space, micro-states and statistical distributions
  - Statistical averages and ergodicity
  - Equilibrium, equilibrium fluctuations, and statistical independence
  - Liouville’s theorem and the concept of statistical ensemble
  - Micro-canonical distribution in classical and quantum statistics
  - Partial equilibrium and the concept of macro-states
  - Entropy, its maximum value and other properties in equilibrium
  - The law of increase of entropy (the second law of thermodynamics)

- **Thermodynamics**
  - Temperature and its positiveness
  - Adiabatic processes and generalized thermodynamic forces
  - Work, heat, and the first law of thermodynamics
  - Thermodynamic potentials, Maxwell & other thermodynamic relations
  - Thermodynamic inequalities and stability criteria
  - Nernst’s theorem and the third law of thermodynamics

- **Canonical & grand-canonical distributions**
  - Canonical Gibbs distribution
  - Maxwell-Boltzmann distribution
  - Monatomic ideal gases
  - Harmonic oscillators
  - The law of equipartition
  - Diatomic and polyatomic gases
  - Grand-canonical Gibbs distribution

- **Quantum statistics**
  - Fermi-Dirac & Bose-Einstein distributions
  - Degenerate Fermi & Bose gases
  - Bose-Einstein condensation
  - Black-body radiation
  - Vibrations of a solid

- **Interacting classical fluids: Short-range inter-particle interactions**
  - Virial expansion and cluster functions
  - Second and third virial coefficients: Hard-sphere, square-well & Lennard-Jones potentials
  - Higher-order virial coefficients
  - Van der Waals equation of state and the liquid-vapor transition

- **Interacting classical fluids: Coulomb interactions**
  - Coulomb interactions & Coulomb fluids in soft matter and biology *(slide presentation)*
  - Primitive model of multi-component Coulomb fluids ("ionic mixtures" or "electrolytes")
  - Mean-field theory: The nonlinear Poisson-Boltzmann equation
  - Linearized mean-field theory
    - Debye-Hückel theory ("Yukawa" plasmas)
    - Break down of the virial expansion & the origin of Debye screening
    - Singular correlation corrections to the equation of state (bulk limiting laws)
  - Electrical double layers: The (mean-field) Gouy-Chapman theory
  - Confined one-component ("counterion-only") Coulomb fluids
    - Counterions at a single charged wall: Mean-field density profile
    - Counterions between two charged walls: Effective counterion-mediated interactions
    - Derjaguin-Landau-Verwey-Overbeek theory of colloidal stability *(slide presentation)*
    - Recent advances in theory & simulations of confined Coulomb fluids *(slide presentation)*

✤ Part II — Equilibrium phase transitions and critical phenomena

- **General aspects and examples** *(blackboard & slide presentation)*
  - Bulk phases and phase transitions in simple fluids & ferromagnets
  - Classifications of (bulk) phase transitions: Thermodynamic non-analyticities
  - Discontinuous (first-order) transitions: Phase separation & coexistence region
  - Continuous (second-order) transitions: Critical points
  - Multicritical points & other typical features of phase diagrams
  - Modern perspective on phase transitions
    - Criticality, correlations, scaling & universality
    - Critical exponents: Experiments, simulations & theory
    - Critical exponents: Thermodynamic inequalities
    - Examples from liquid crystals to lipids to superfluids to superconductors
    - Order parameter & broken symmetry
- **Ising model**
  - Ising model in one dimension
    - Exact transfer matrix solution, spin correlations & correlation length
    - Kinks & the absence of finite-T spontaneous magnetization
    - Nature of the singularity & phase transition at $T = 0$
  - Ising model in two dimensions
    - Domain walls & the existence of finite-T phase transition
    - Critical exponents (Onsager's exact results)
  - Mean-field theory in arbitrary dimension
    - Weiss molecular-field theory
    - Bragg-Williams (variational) approximation
    - Infinite-range (or infinite-dimensional) Ising model
    - Mean-field phase diagrams: First- & second-order transitions
  - Ising critical exponents: Mean field *vs* exact *vs* simulation results
- **Landau mean-field theory**
  - Landau-Ginzburg phenomenology
  - ϕ^4 theory
    - Spontaneous symmetry breaking
    - Thermodynamic limit & the ergodicity breaking
    - Critical exponents: Mean-field Ising universality class
  - ϕ^3 theory: Continuous *vs* discontinuous transition
  - ϕ^6 theory: Tricritical point *(included in Homework #8)*

- Liquid-vapor transition revisited
  - Virial expansion & the van der Waals equation of state
  - Maxwell construction, phase coexistence & the critical point
  - Comparison with ϕ^4 theory of ferromagnets
- Ginzburg-Landau theory of superconductivity *(included in Homework #8)*

Gaussian-fluctuation (one-loop) corrections

- Landau-Ginzburg-Wilson Hamiltonian
  - Field fluctuations & stiffness
  - Correlation functions & susceptibility
  - Modified singularities: Free energy & heat capacity
  - Liquid-vapor system: Critical opalescence
- Upper critical dimension & the Ginzburg criterion
- Dimensional analysis & anomalous dimensions
- Coupling to gauge fields: Anderson-Higgs mechanism (*included in Homework #9)*

Widom scaling

- Homogeneous functions
- Widom scaling hypothesis
- Critical exponents: Scaling laws
- Spatial scale-invariance & hyperscaling laws
- Hyperscaling above dimension four: A paradox?

Real-space renormalization group (RG) transformation

- Kadanoff block spins & derivation of Widom scaling
- Wilson block spins, fixed points & calculation of critical exponents
- General properties of RG flows & some characteristic fixed points
- Ising model on a triangular lattice: Real-space RG in two dimensions

Part III — Non-equilibrium Statistical Physics

Kinetic theory & Boltzmann equation

Dissipative (Brownian) dynamics of particles

- Random walks and Brownian motion
- Elements of the theory of stochastic processes
- Langevin equation: From phenomenological to formal derivation
- (Smoluchowski-) Fokker-Planck equation