Course on "Statistical Physics of Fields In and Out of Equilibrium"

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◉ **Further information about this course, including problem sets and term-essay topics, are available at:** **http://physics.ipm.ac.ir/~naji/fields_SS2016.html**

◉ **Subjects covered in this course:**

Statistical Physics of Fields In Equilibrium

✤ **Part I — Preliminaries**

- **Statistical physics and thermodynamics of many-particle systems**
  - A conceptual review in four lectures based on the classic text by Landau & Lifshitz (Vol. 5)
- **Statistical fields: A scenery** *(slide presentation)*
  - Spin & particle models: From classical to modern examples
  - Modern applications of statistical field theory in soft & condensed matter physics
- **Statistical field theory: Basic methods**
  - Construction of continuum field actions
    - Coarse-graining of microscopic Hamiltonians
    - Symmetries & phenomenological actions (or effective 'Hamiltonians')
  - Classical field theory
    - Calculus of variations
    - The least-action principle & Euler-Lagrange equations
    - Continuous symmetries & Noether's theorem
    - Energy-momentum (or stress-energy) tensor
  - Fluctuating fields & functional-integral methods
    - Path integrals: An overview
    - Functional integrals & the partition function
    - Generating functionals & connected correlation (Green) functions
    - Response functions & the fluctuation-dissipation theorem
    - Gaussian integrals & Wick's theorem ('free field theory')
    - Formal perturbative expansion of field interaction
    - Feynman diagrams
  - Other routes to constructing field theories
    - A modern (re)formulation of the Hubbard-Stratonovich transformation
    - Construction of exact field theories: From $O(n)$ model to Coulomb fluids
    - Edwards-Gupta method: Effective field theory for Ising & $O(n)$ models
  - Saddle-point method
    - Mean field theory as a steepest descent (saddle-point) approximation
    - Loop expansion & Gaussian-fluctuation (one-loop) corrections
    - Saddle-point approximation in the presence of zero modes (*included in Homework #15*)
  - Other useful techniques (applicable to **Part II**)
    - Schwinger-Dyson equations
    - Ward identities *(included in Homework #7)*

- Functional determinants (applicable to **Part III**)
  - Van Vleck-Pauli-Morette (VVPM) formula
  - Spectral ζ-functions & heat kernel: An overview
  - ζ-function regularization
  - Gel'fand-Yaglom (GY) formula: Contour integration approach
  - Coleman's proof of GY formula *(included in Homework #9)*
  - Equivalence of VVPM & GY formulas *(included in Homework #9)*
- Zero modes (applicable to **Part IV**) (*included in Homework #15*)
  - Continuous symmetry & Goldstone modes
  - Gaussian-fluctuation (one-loop) corrections in the presence of zero modes
  - Perturbative expansion in the presence of zero modes

✤ Part II — Field theory for Coulomb fluids (or "Coulomb gases")

‣ Coulomb interactions & Coulomb fluids in soft matter and biology *(slide presentation)*

‣ Coulomb fluids: General formalism

- Exact field action *via* Hubbard-Stratonovich transformation

‣ Mean-field theory

- Multi-component Coulomb fluids ("ionic mixtures" or "electrolytes")
  - Saddle-point approximation: The nonlinear Poisson-Boltzmann equation
  - Electrical double layers: The Gouy-Chapman theory
  - Linearization approximation: The Debye-Hückel theory
  - Generalized electrostatic stress tensor *(not covered in this Course)*
- Confined one-component ("counterion-only") Coulomb fluids
  - Counterions at a single charged wall: Mean-field density profile
  - Counterions between two charged walls: Effective counterion-mediated interactions

‣ Beyond mean-field theory (I): Weak-coupling regime

- Bulk ionic mixtures ("Yukawa" plasmas)
  - Quadratic Debye-Hückel field action
  - Break down of the virial expansion & the origin of Debye screening
  - Singular correlation corrections to the equation of state (bulk limiting laws)
- Confined one-component ("counterion-only") Coulomb fluids: One-loop corrections
  - Fluctuation corrections to counterion density: One & two walls *(included in Homework #8)*
  - Gaussian-fluctuation corrections to counterion-mediated interactions between two walls

‣ Beyond mean-field theory (II): Strong-coupling regime *(slide presentation)*

- Weak *vs* strong couplings: A unified field-theoretic approach
- Strong-coupling theory: Virial expansion restored for confined Coulomb fluids
- Recent advances in the theory & simulations of confined Coulomb fluids

✤ Part III — Fluctuation-induced phenomena (Casimir & pseudo-Casimir effects)

‣ A plethora of fluctuation-induced phenomena *(slide presentation)*

- Examples from quantum vacuum fluctuations to superfluid films to liquid crystals to Derjaguin-Landau-Verwey-Overbeek theory of colloidal stability

‣ Quantum vacuum & electromagnetic field fluctuations

- Non-retarded Casimir forces: Perfect "mirrors" at zero temperature
- Thermal (classical) Casimir forces: Massless fields at high temperature
- Lifshitz theory for dielectrics at finite temperature: Retardation effects

‣ Fluctuation-induced effects in correlated liquids

- Pseudo-Casimir forces in superfluid helium films
- Pseudo-Casimir forces in nematic liquid-crystalline films *(included in Homework #10)*

- **Fluctuation-induced effects in Coulomb fluids**
  - Massive field fluctuations & Pseudo-Casimir effect in "Yukawa" plasmas
  - Fluctuations on a non-uniform background: Confined Coulomb fluids revisited

✤ Part IV — Phase transitions and critical phenomena

- **General aspects and examples** *(blackboard & slide presentation)*
  - Bulk phases and phase transitions in simple fluids & ferromagnets
  - Classifications of (bulk) phase transitions: Thermodynamic non-analyticities
  - Discontinuous (first-order) transitions: Phase separation & coexistence region
  - Continuous (second-order) transitions: Critical points
  - Multicritical points & other typical features of phase diagrams
  - Modern perspective on phase transitions
    - Criticality, correlations, scaling & universality
    - Critical exponents: Experiments, simulations & theory
    - Critical exponents: Thermodynamic inequalities
    - Examples from liquid crystals to lipids to superfluids to superconductors
    - Order parameter & broken symmetry

- **Ising model**
  - Ising model in one dimension
    - Exact transfer matrix solution, spin correlations & correlation length
    - Kinks & the absence of finite-T spontaneous magnetization
    - Nature of the singularity & phase transition at $T = 0$
  - Ising model in two dimensions
    - Domain walls & the existence of finite-T phase transition
    - Critical exponents (Onsager's exact results)
  - Mean-field theory in arbitrary dimension
    - Weiss molecular-field theory
    - Bragg-Williams (variational) approximation
    - Infinite-range (or infinite-dimensional) Ising model
    - Mean-field phase diagrams: First- & second-order transitions
  - Ising critical exponents: Mean field *vs* exact *vs* simulation results
  - Yang-Lee theory of phase transitions *(not covered in this Course)*

- **Landau mean-field theory**
  - Landau-Ginzburg phenomenology
  - ϕ^4 theory
    - Spontaneous symmetry breaking
    - Thermodynamic limit & the ergodicity breaking
    - Critical exponents: Mean-field Ising universality class
  - ϕ^3 theory: Continuous *vs* discontinuous transition
  - ϕ^6 theory: Tricritical point *(included in Homework #11)*
  - Liquid-vapor transition
    - Virial expansion & the van der Waals equation of state
    - Maxwell construction, phase coexistence & the critical point
    - Comparison with ϕ^4 theory of ferromagnets
  - Ginzburg-Landau theory of superconductivity *(included in Homework #11)*

- **Gaussian-fluctuation (one-loop) corrections**
  - Landau-Ginzburg-Wilson Hamiltonian
    - Field fluctuations & stiffness
    - Correlation functions & susceptibility
    - Modified singularities: Free energy & heat capacity

    - Liquid-vapor system: Critical opalescence
  - Upper critical dimension & the Ginzburg criterion
  - Dimensional analysis & anomalous dimensions

- **Widom scaling**
  - Homogeneous functions
  - Widom scaling hypothesis
  - Critical exponents: Scaling laws
  - Spatial scale-invariance & hyperscaling laws
  - Hyperscaling above dimension four: A paradox?

- **Real-space renormalization group (RG) transformation**
  - Kadanoff block spins & derivation of Widom scaling
  - Wilson block spins, fixed points & calculation of critical exponents
  - General properties of RG flows & some characteristic fixed points
  - Ising model on a triangular lattice: Real-space RG in two dimensions

- **Momentum-space RG transformation**
  - RG scheme revisited: Coarse grain, rescale & renormalize
  - Gaussian model: Exact solution & RG analysis
  - Dangerous irrelevant variables: Hyperscaling revisited

- **ϕ^4 theory**
  - Perturbative RG & Feynman diagrams
  - ϵ-expansion & critical exponents to order $O(\epsilon)$
  - Supplementary remarks
    - Finite anomalous dimension to order $O(\epsilon^2)$
    - Asymptotic behavior of ϵ-expansion
    - Results from resummation techniques
    - Ising universality class: Comparison with experiments & simulations

- **Continuous symmetry**
  - Discrete symmetry breaking: Domain walls revisited
  - Continuous symmetry breaking
    - Goldstone modes: Examples from crystalline solids to (Heisenberg) ferromagnets to liquid crystals to superfluid Helium
    - Goldstone's theorem revisited
    - Massless fluctuations & the lower critical dimension
    - Mermin-Wagner (-Hohenberg-Coleman) theorem
  - Coupling to gauge fields: Anderson-Higgs mechanism (*included in Homework #12)*

- **Other subjects** *(not covered in this Course / potential subjects for term essays)*
  - Nonlinear σ model: Critical behavior near dimension two
  - XY model, topological defects & the Kosterlitz–Thouless transition
  - RG for the two-dimensional Coulomb gas
  - Two-dimensional solids & melting

Statistical Physics of Fields Out of Equilibrium

✤ **Part V — Dissipative dynamics of fields near and far from equilibrium**

- **Dissipative (Brownian) dynamics of particles**
  - Elements of the theory of stochastic processes
  - Langevin equation: From phenomenological to formal derivation
  - (Smoluchowski-) Fokker-Planck equation
  - Path-integral formulation of Brownian motion

- **Dissipative dynamics of fields: A scenery** *(slide presentation)*
  - Models of dissipative field dynamics: Examples from nucleation & spinodal decomposition to elastic membranes & growing surfaces to active fluids & bacterial suspensions
- **Critical dynamics & dynamic scale-invariance near equilibrium**
  - Non-conserved dynamics (Model A: Stochastic time-dependent Landau-Ginzburg equation)
  - Conserved dynamics (Model B: Stochastic Cahn–Hilliard equation)
  - An overview of dynamic perturbation theory
- **Critical dynamics & dynamic scale-invariance far from equilibrium** *(slide presentation)*
  - Kardar-Parisi-Zhang equation
  - Dynamic RG at a glance: The KPZ universality class
- **Field theory for stochastic partial differential equations**
  - Langevin-type partial differential equations
  - Generalized (Smoluchowski-) Fokker-Planck equation *(included in Homework #16)*
  - Basics of functional-integral representation & the Martin-Siggia-Rose action