Course on "Statistical Physics of Fields In and Out of Equilibrium"
Spring Semester (2016)
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Homework #13: Phase Transitions & Critical Phenomena: Real-space RG Due: July 20, 2016

Note: *References from which some of the following problems and figures have been taken are available upon request.*

1: *Ising model on square and honeycomb lattice.* Apply Kadanoff's block-spin construction and Wilson's real-space RG transformation to the two-dimensional, nearest-neighbor, spin-1/2 Ising model on (i) square and (ii) honeycomb lattices and find the critical exponents of these models within the first-order cumulant approximation. Compare the critical values of the exchange coupling constant obtained from your calculation with the exact results given (in units of k_BT) by $K_c \simeq 0.441$ for the square lattice and $K_c \simeq 0.658$ for the honeycomb lattice. Compare your results with those obtained in class for the case of a two-dimensional triangular lattice.

2: *Exact renormalization on hierarchical Berker lattice.* Consider the nearest-neighbor, spin-1/2 Ising model on a two-dimensional hierarchical lattice constructed according to the steps shown in the figure below. Apply a real-space RG transformation to this model and calculate the RG flow and its fixed points in the (K, B) plane, with K being the exchange coupling constant and B the external magnetic field (both in units of k_BT). Show that the renormalization group theory of phase transition on this lattice is *exact!*

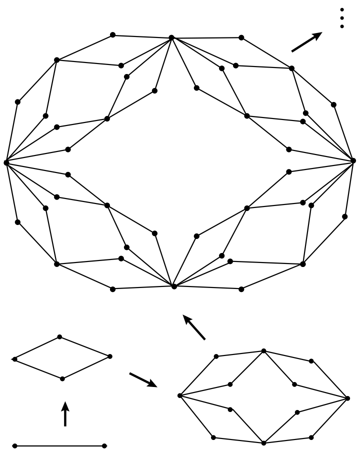

Figure 1. (See Problem 2)

3: *Ising model on a fractal.* Consider the nearest-neighbor, spin-1/2 Ising model on the fractal set known as the *Sierpinski gasket (or triangle)* (see the figure below and `https://en.wikipedia.org/wiki/Sierpinski_triangle` for further information). Find the renormalization flow for the coupling constant and its corresponding fixed points. Discuss the ordering types and the phase transitions possible in this system.

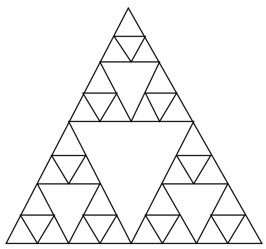

Figure 2. (See Problem 3)