Course on "Advanced Statistical Physics"

School of Nano Science, Institute for Research in Fundamental Sciences (IPM), Tehran, Iran
Spring Semester 2016 (1394-95)

Lecturer: Ali Naji (School of Physics, IPM)

Office: Room 503, Farmanieh Central Building (IPM)
Email: a.naji@ipm.ir; Tel: +98-21-22280692 ext. 3039

◉ Outline of the course:

✤ Part I — Equilibrium Statistical Physics

▸ Foundations: Closed (and quasi-closed) systems

- Phase space, micro-states and statistical distributions
- Statistical averages and ergodicity
- Equilibrium, equilibrium fluctuations, and statistical independence
- Liouville's theorem and the concept of statistical ensemble
- Microcanonical distribution for classical and quantum statistics
- Partial equilibrium and the concept of macro-states
- Entropy and its properties in equilibrium
- The law of increase of entropy (the second law of thermodynamics)

▸ Thermodynamics

- Temperature and its positiveness
- Adiabatic processes and generalized thermodynamic forces
- Work, heat, and the first law of thermodynamics
- Thermodynamic potentials
- Maxwell and other thermodynamic relations
- Thermodynamic inequalities and stability criteria
- Nernst's theorem and the third law of thermodynamics

▸ Canonical & grand-canonical distributions

- Canonical Gibbs distribution
- Maxwell-Boltzmann distribution
- Monatomic ideal gases
- Harmonic oscillators
- Law of equipartition
- Diatomic and polyatomic gases
- Grand-canonical Gibbs distribution

▸ Quantum statistics

- Fermi-Dirac & Bose-Einstein distributions
- Degenerate Fermi gas
- Degenerate Bose gas
- Bose-Einstein condensation

- Black-body radiation
- Vibrations of a solid

- **Interacting classical fluids I**
  - Thermodynamics and the radial distribution function
  - Virial expansion and cluster functions
  - Second and third virial coefficients
    - Hard-sphere, square-well & Lennard-Jones potentials
  - Higher-order virial coefficients
  - Van der Waals equation of state and liquid-vapor transition
  - Special topics *(if time permits)*
    - Static structure factor
    - Yvon–Born–Green hierarchy
    - Integral equations and hypernetted-chain approximation
    - Ornstein-Zernike equation
    - Percus-Yevick equation
    - Mean spherical approximation
- **Interacting classical fluids II — Coulomb fluids**
  - Coulomb interactions in soft matter and biology *(slides presentation)*
  - Coulomb fluids: The primitive model
  - Mean-field theory: The Poisson-Boltzmann equation
  - Debye-Huckel approximation
  - Limiting laws and correlation correction for bulk electrolytes
  - Electrical double layer: The Gouy-Chapman (mean-field) theory
  - Colloidal interactions *(slides presentation)*
    - DLVO theory
    - Confined Coulomb fluids — from weak fluctuations to strong couplings
    - Recent advances in theory and simulations

✤ **Part II — Equilibrium phase transitions and critical phenomena**

- **Generalities, brief history and modern examples** *(slides presentation)*
  - Bulk phases and phase transitions: Simple fluids & ferromagnets
  - Criticality, scaling & universality: Experiments and simulations
  - Classifications of (bulk) phase transitions
  - Multicritical points & other features of phase diagrams
  - Order parameter & broken symmetry
  - Modern examples from liquid crystals to superfluids & superconductors
  - Spin and particle models: From classical to modern examples
- **Widom scaling hypothesis**
  - Homogeneous functions
  - Scale invariance & scaling exponents

- **Ising model**
  - Exact solutions
  - Mean-field theory: Weiss molecular field & Bragg-Williams approximation
  - Kadanoff block spins & Widom scaling
- **Real-space renormalization group (RG)**
  - Wilson block spins, renormalization & fixed points
  - Ising model revisited: RG in one and two dimensions
- **Landau mean-field theory**
  - Landau-Ginzburg phenomenology
  - ϕ^4 theory: Spontaneous symmetry breaking
  - Thermodynamic limit and ergodicity breaking
  - Discontinuous *vs* continuous transition (first *vs* second order)
  - ϕ^6 theory: Tricritical point & other aspects
- **Gaussian fluctuations**
  - Correlation functions & susceptibilities
  - Fluctuation corrections to mean field & the upper critical dimension
  - Ginzburg criterion
  - Discrete symmetry breaking: Domain walls
  - Continuous symmetry breaking: Goldstone modes
  - Lower critical dimension & the Mermin-Wagner theorem

✤ **Part III — Non-equilibrium Statistical Physics**

- **Kinetic Theory & Hydrodynamics**
  - Boltzmann equation
  - Hydrodynamic limit: The Navier-Stokes equations
  - Special topics *(if time permits)*
    - Hydrodynamic fluctuations near equilibrium
    - Linearized hydrodynamics and hydrodynamic collective modes
    - Transport coefficients and Onsager's relations
    - Linear response theory and the fluctuation-dissipation theorem
- **Dissipative dynamics of particles**
  - Random walks and Brownian motion
  - Theory of stochastic processes: A brief review
  - Langevin equation
  - Fokker-Planck equation